Engineering Notes

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Fixed-Order Compensator Design Based on Frequency-Shaped Cost Functionals

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Introduction

FREQUENCY-SHAPED cost functionals have been introduced as a way of embedding classical design concepts within the context of LQ optimal control.[1,2] This procedure has been demonstrated as an effective approach to active damping of structural modes, rejecting disturbances occurring over a narrow frequency range, and improving robustness to plant uncertainty. When applied to the design of observer-based compensators, the resulting controller unfortunately is dimensionally larger than the system model because the realization of the frequency-shaping dynamics is an integral part of the overall compensator design process.[3]

In this Note, we consider the use of frequency-shaped cost functionals in the context of LQ optimal output feedback design of fixed-order compensators. The formulation exploits an observer canonical form to represent the compensator dynamics and thus avoids the problem of overparameterization that has appeared in earlier formulations.[4] The major advantages of the design approach are that the order of the compensator is fixed by the design process, and that it is not necessary to realize the frequency-shaping dynamics as an integral part of the compensator design. This avoids the two-step approach of full-state feedback design followed by the design of an observer, which in the case of a full-order observer requires a compensator of order $n + n_f$ where n is the order of the plant and n_f is the order of frequency-shaping realization. The formulation also precludes the use of direct feedback of the plant output. Thus, it carries the same advantages of a full-order observer in reducing the effect of sensor noise and improving the robustness to high-frequency unmodeled dynamics by guaranteeing an additional 20 dB/decade roll-off (over that of the open loop plant) at high frequencies.

A structural vibration model is used to illustrate the design procedure. A numerically convergent algorithm[5] is used to solve for the optimal compensator parameters.

Fixed-Order Dynamic Compensation

It has been shown[6] that for a multivariable system described by

$$\dot{x} = Ax + Bu, \qquad x \in R^n \tag{1}$$

$$y = Cx, \qquad y \in R^p \tag{2}$$

a fixed-order compensator without direct feedthrough of the output can be formulated in observer canonical form as

$$u = -H^\circ z, \qquad u \in R^m \tag{3}$$

$$\dot{z} = P^\circ z + u_c, \qquad z \in R^{n_c} \tag{4}$$

$$u_c = P_z u - Ny, \qquad u_c \in R^{n_c} \tag{5}$$

where

$$H^\circ = \text{block diag}\,\{[0 \ldots 0\ 1]_{1 x \nu_i}\, i = 1, \ldots, m\} \tag{6}$$

$$P^\circ = \text{block diag}\,[P_1^\circ, \ldots, P_m^\circ] \tag{7}$$

$$P_i^\circ = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{\nu_i x \nu_i} \tag{8}$$

In Eqs. (3-5), N and P_z are free parameter matrices with dimensions $(n_c \times p)$ and $(n_c \times m)$, respectively. The dimensions of H° and P° are defined by the observability indices of the compensator, which are chosen to satisfy

$$\sum_{i=1}^{m} \nu_i = n_c, \qquad \nu_i \le \nu_{i+1}$$

The augmented system matrices

$$\tilde{A} = \begin{bmatrix} A & -BH^\circ \\ 0 & P^\circ \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix} \tag{9}$$

$$\tilde{C} = \begin{bmatrix} C & 0 \\ 0 & H^\circ \end{bmatrix}, \qquad \tilde{G} = [\,N \quad P_z\,] \tag{10}$$

define an optimal output feedback problem, with the quadratic performance index

$$J = E_{x_0}\left\{\int_0^\infty [x^t Q x + u_c^t R u_c]\,\mathrm{d}t\right\} \tag{11}$$

where the augmented state vector is

$$\tilde{x}^t = [x^t z^t] \tag{12}$$

and the control is defined as

$$u_c = -\tilde{G}\,\tilde{C}\,\tilde{x} \tag{13}$$

Frequency Shaping

It is well known that frequency-shaped cost functionals can be used to damp structural modes, improve robustness, and reject external disturbances acting over a narrow frequency range. However, papers on this subject have required that the frequency shaping be realized as part of the compensator design.[3,6,7] We present here an approach to frequency shaping

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that does not increase the order of the compensator. The idea is to adopt the following performance index:

$$J = E_{x_0}\left\{\int_0^\infty [x^tQx + y_2^t y_2 + u_c^t R u_c]\,\mathrm{d}t\right\} \tag{14}$$

where y_2 is defined by

$$\dot{w} = Fw + My_1, \qquad y_1 = C_1 x \tag{15}$$

$$y_2 = Ew + Jy_1 \tag{16}$$

That is, y_2 is the output of a filter driven by a suitably chosen linear combination of the plant states. The augmented system matrices become

$$\tilde{A} = \begin{bmatrix} A & 0 & -BH^\circ \\ MC_1 & F & 0 \\ 0 & 0 & P^\circ \end{bmatrix} \tag{17}$$

$$\tilde{B} = \begin{bmatrix} 0 \\ 0 \\ I_{nc} \end{bmatrix}, \qquad \tilde{C} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & H^\circ \end{bmatrix} \tag{18}$$

where now $\tilde{x}^t = [x^t w^t z^t]$. The resulting state weighting matrix, when the performance index is reformulated in the form of Eq. (11), becomes

$$\tilde{Q} = \begin{bmatrix} Q + C_1^t J^t J C_1 & C_1^t J^t E & 0 \\ E^t J C_1 & E^t E & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

and u_c remains as defined in Eq. (13).

Numerical Results

We present in this section an example to illustrate the design procedure. This example treats a structural vibration model that arises in considering the fast dynamics of a lightweight flexible arm moving along a predefined path.[8] For full-state feedback design, the derivatives of the deflections need to be estimated since only the deflections are available as outputs from strain gage measurements. Rather than design an observer, a second-order compensator is designed to damp the first two structural modes without direct feedthrough of the outputs. The main purpose of the example is to illustrate how the design requirements can be easily related to parameters in the performance index. The solutions were obtained using the algorithm described in Ref. 5, for which convergence is guaranteed subject to a set of mild restrictions on the problem formulation.

The flexible dynamics and outputs are defined by the following system matrices:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -205.4 & -1900 & 0 & 0 \\ -053.0 & -3051 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ -2.33 \\ -0.75 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{20}$$

The state variables are $x^T = [\delta_1, \delta_2, \dot{\delta}_1, \dot{\delta}_2]$, where strain gage measurements of δ_1 and δ_2 represent deflections of the arm at the endpoint and midpoint, respectively. The open loop modes are at $\pm\,\omega_i j$, where $\omega_1 = 13.77$ and $\omega_2 = 89.8$. The frequency-

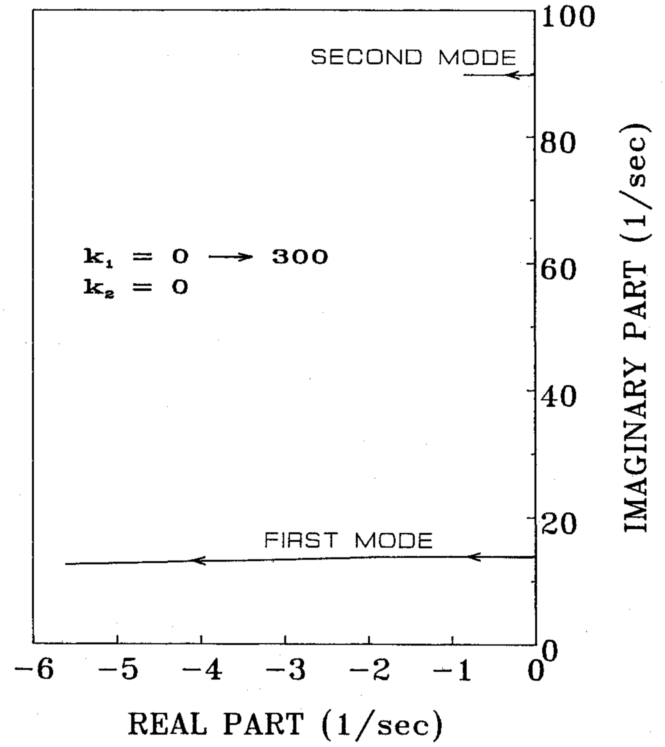

Fig. 1 Effect of frequency shaping as k_1 is increased from 0 to 300.

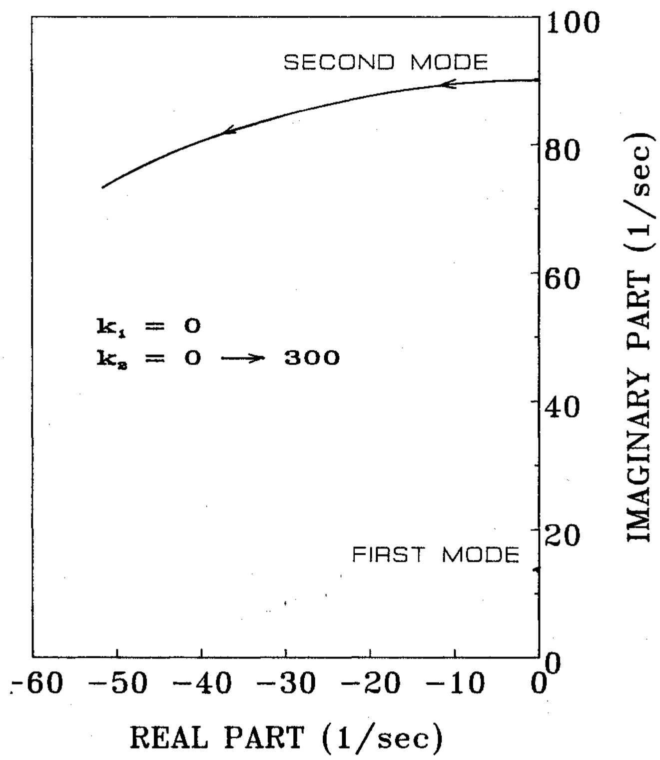

Fig. 2 Effect of frequency shaping as k_2 is increased from 0 to 300.

shaping dynamics in Eqs. (15) and (16) were defined as the realizations of the transfer functions

$$y_{1_i}/\dot{\delta}_2 = k_i \frac{s^2/\omega_i + 2\zeta_d s/\omega_i + 1}{s^2/\omega_i + 2\zeta s/\omega_i + 1} \qquad i = 1, 2 \tag{21}$$

where $\zeta_d = 0.7$, $\zeta = 0.01$. This, in effect, amplifies the weightings on the plant state $\dot{\delta}_2$ in the vicinity of ω_i. The weightings on each mode are independently controlled by selecting the weighting parameters k_i. A second-order compensator (n_c

= 2) was designed with $Q = 0$, $R = 0.1$ in Eq. (14). The parameters k_i were adjusted to achieve the desired damping in the structural modes.

Experience with the solution procedure showed that the damping on the closed-loop structural modes could be individually adjusted by the choice of k_i. Moreover, the damping can be introduced with only minor change in natural frequency. Figure 1 illustrates the effect that increasing k_1 has on the modal damping, with $k_2 = 0$. Note that this results in increased damping on the first mode, whereas the second mode remains relatively unaffected. Figure 2 illustrates the fact that increasing k_2 with $k_1 = 0$ has the opposite effect. In both cases, the compensator introduces two additional closed-loop, low-frequency poles that are almost unobservable in the plant states.

For $k_1 = 350$, $k_2 = 345$, the closed-loop structural mode dampings were $\zeta_1 = 0.52$ and $\zeta_2 = 0.70$, respectively. Attempts to increase the damping further resulted in convergence difficulties with the numerical algorithm used to find the optimal G. Normally, the algorithm converged in fewer than 10 iterations.

The final solution was

$$G = \left[\begin{array}{cc|c} -54.3 & -628.5 & 0.687 \\ -922.0 & -1.19 \times 10^6 & 126.8 \end{array}\right] \tag{22}$$

Conclusions

A method for designing fixed-order compensators using frequency-shaped cost functionals has been outlined. The major advantages are that the order of the compensator is fixed by the design process, and it is not necessary to realize the frequency-shaping dynamics as an integral part of the compensator design. The example illustrates the use of the design procedure to damp the fast structural modes of a flexible arm.

Acknowledgment

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Effects of Atmospheric Density Gradient on Control of Tethered Subsatellites

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Nomenclature

$[A]$, a_i	= plant matrix, see Eqs. (17), (21), and (24)
$[B]$	= input matrix, see Eqs. (17) and (22)
C_D	= aerodynamic drag coefficient
D	= aerodynamic drag force
$K_\delta, K_{\delta'}, K_\gamma, K_{\gamma'}$	= control gains, see Eq. (25)
l	= length of tether
m	= mass of subsatellite
$[Q]$	= weighting matrix of the state, see Eq. (26)
$\{q\}$	= state vector, see Eq. (18)
R	= radius of the orbit of main satellite
R_u	= weighting coefficient of the control force, see Eq. (26)
S	= equivalent drag area (involving both subsatellite and tether)
T	= tension in tether
t	= time
u	= normalized control force, see Eq. (23)
v	= velocity of subsatellite relative to the atmosphere
α	= angle between the local horizon and relative velocity vector, see Fig. 1
γ, δ	= normalized infinitesimal displacements of subsatellite from the steady-state equilibrium point, see Eqs. (19) and (20)
θ	= pitch angle between local vertical and tether line, see Fig. 1
λ	= coefficient of the atmospheric density gradient, see Eq. (16)
μ	= see Eq. (15)
ξ	= nondimensional length of tether, see Eq. (8)
ρ	= density of the atmosphere
τ	= nondimensional time, see Eq. (10)
Ω	= rotational angular rate of the Earth
ω'	= orbital angular rate

Subscript

s	= steady-state solution

I. Introduction

TETHERED subsatellites deployed in low-altitude orbits have been proposed as means of upper atmospheric experiments. Many works on the dynamics and control of the tethered subsatellites have been reported.[1] Beletskii and Levin[2] and Onoda and Watanabe[3] have shown that the atmospheric density gradient (together with the orbital angular velocity and the elasticity of the tether) unstabilizes the in-plane swinging motion of uncontrolled tethered subsatellites in the state of station keeping.

In this Note, the effects of the atmospheric density gradient on the swinging motion of tension-controlled tethered subsatellites and on the design of the control systems are investigated based on a simple model.

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